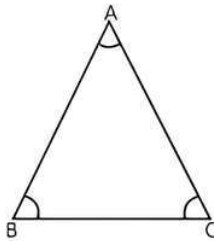


7

Triangles

Fastrack Revision

- **Triangle:** It is closed figure formed by three intersecting lines. A triangle has three sides, three angles and three vertices. In $\triangle ABC$, AB, BC and CA are sides; $\angle A$, $\angle B$ and $\angle C$ are angles; A, B and C are vertices.



- **Congruence of Triangles:** Congruent means equal in all aspects or the figures with same shapes and sizes. The two triangles are said to be congruent, if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle. Congruence is denoted by the symbol \cong .

- **Criteria for Congruence of Triangles:**

- SSS (Side-Side-Side) Congruence:** If three sides of a triangle are equal to the three sides of another triangle, then the two triangles satisfy the SSS congruency.
- SAS (Side-Angle-Side) Congruence:** If two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then the two triangles satisfy the SAS congruency.
- ASA (Angle-Side-Angle) Congruence:** If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the triangles satisfy the ASA congruency.
- RHS (Right-Hand-Side) Congruence:** If the hypotenuse and one side of one triangle are

equal to the hypotenuse and one side of the other triangle, then the two triangles satisfy the RHS congruency.

- **Some Properties of a Triangle:**

- The sum of three angles of a triangle is 180° .
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is of 60° .
- The altitude drawn from vertex, it bisects perpendicularly the base of an equilateral and isosceles triangle.

Knowledge BOOSTER

- If $\triangle ABC$ and $\triangle PQR$ are congruent, then we have to be careful for writing in symbol i.e., $\triangle ABC \cong \triangle PQR$, hence we do not write as $\triangle ABC \cong \triangle QPR$ or $\triangle ACB \cong \triangle PQR$ etc.
- CPCT is a short form of writing corresponding parts of congruent triangles.
- Two geometric figures are said to be congruent, if they are equal in all respects.
- Every triangle is congruent to itself.
- If any two pairs of angles and one pair of corresponding sides are equal, then the two triangles are congruent. This may be called as AAS congruence rule.
- The medians of an equilateral triangle are equal.



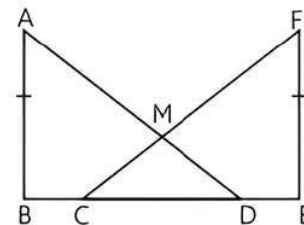
Practice Exercise



Multiple Choice Questions

- Q 1. Which of the following is not a criterion for congruence of triangles?
- a. SAS b. ASA c. SSA d. SSS
- Q 2. If $AB = QR$, $BC = RP$ and $CA = PQ$, then:
- a. $\triangle ABC \cong \triangle PQR$ b. $\triangle CBA \cong \triangle PRQ$
 c. $\triangle BAC \cong \triangle RPQ$ d. $\triangle PQR \cong \triangle BCA$

- Q 3. In the given figure, $AB \perp BE$ and $EF \perp BE$. Also $BC = DE$ and $AB = EF$. Then:



- a. $\triangle ABD \cong \triangle EFC$ b. $\triangle ABD \cong \triangle CEF$
 c. $\triangle ABD \cong \triangle FEC$ d. $\triangle ABD \cong \triangle CMD$

- Q 4. It is given that $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle A + \angle B + \angle C = 180^\circ$, $\angle B = 40^\circ$ and $\angle A = 80^\circ$. Then which of the following is true?
- $DF = 5$ cm, $\angle F = 60^\circ$
 - $DF = 5$ cm, $\angle E = 60^\circ$
 - $DE = 5$ cm, $\angle E = 60^\circ$
 - $DE = 5$ cm, $\angle D = 40^\circ$

- Q 5. In triangles ABC and DEF , $AB = FD$ and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if:

- $BC = EF$
- $AC = DE$
- $AC = EF$
- $BC = DE$

- Q 6. If $\triangle ABC$; D , E and F are the mid-points of BC , CA and AB respectively; then $\triangle DEF$ is not congruent to the following triangle:

- ABC
- BDF
- AFE
- CDE

- Q 7. In $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$. Then $\angle C$ is equal to:

- 40°
- 50°
- 80°
- 130°

- Q 8. In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ is equal to:

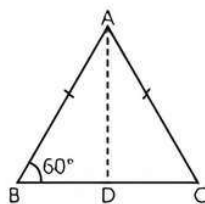
- 80°
- 40°
- 50°
- 100°

- Q 9. In $\triangle PQR$, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm. Then the length of PQ is:

- 4 cm
- 5 cm
- 2 cm
- 2.5 cm

- Q 10. In the given figure, $AB = AC$, AD is the median and its bisect $\angle A$, then $\angle BAD$ is equal to: (use $\angle A + \angle B + \angle C = 180^\circ$)

- 15°
- 30°
- 20°
- 40°

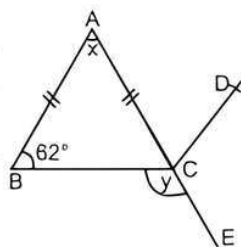


- Q 11. In triangles ABC and PQR , $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are:

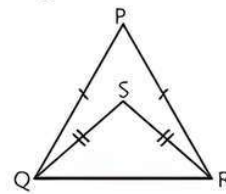
- isosceles but not congruent
- isosceles and congruent
- congruent but not isosceles
- neither congruent nor isosceles

- Q 12. In the adjoining figure, ABC is an isosceles triangle whose side AC is produced to E . Through C , CD is drawn parallel to BA . The value of x is: (Use the sum of all angles of a triangle is 180°).

- 56°
- 120°
- 122°
- 76°

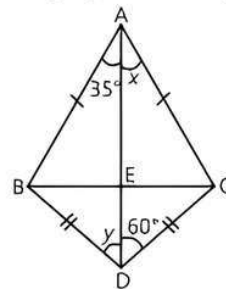


- Q 13. In the given figure, the ratio of $\angle PQS : \angle PRS$ is:



- 1 : 1
- 1 : 2
- 2 : 1
- 1 : 3

- Q 14. In the adjoining figure, x and y are:



- $x = 30^\circ$, $y = 65^\circ$
- $x = 35^\circ$, $y = 60^\circ$
- $x = 35^\circ$, $y = 65^\circ$
- $x = y = 45^\circ$



Assertion & Reason Type Questions

Directions (Q. Nos. 15-19): In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:

- Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
- Assertion (A) is true but Reason (R) is false.
- Assertion (A) is false but Reason (R) is true.

- Q 15. Assertion (A): If we draw two triangles with angles 40° , 60° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle, then two triangles are not congruent.

Reason (R): If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

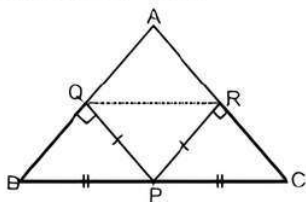
- Q 16. Assertion (A): In triangles ABC and PQR , if $\angle A = \angle P$, $\angle C = \angle R$ and $AC = PR$, then the two triangles are congruent by ASA congruence.

Reason (R): If two angles and included side of a triangle are equal to the corresponding angles and side of the other triangle, then the triangles are congruent by ASA congruence.

Q 17. Assertion (A): In $\triangle PQR$, $PQ = QR$ and $\angle R = 75^\circ$, then $\angle P$ is 52° .

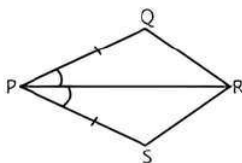
Reason (R): In a triangle, angles opposite to equal sides are equal.

Q 18. Assertion (A): In $\triangle ABC$, P is the mid-point of BC. If $PQ \perp AB$ and $PR \perp AC$, such that $PQ = PR$, then $BQ = CR$.



Reason (R): If two angles and the included side of one triangle are equal to two angles and the included side of other triangle, then two triangles are congruent by ASA rule.

Q 19. Assertion (A): In a quadrilateral PQRS, $PQ = PS$ and PR bisects $\angle P$ by SAS congruence rule.



Reason (R): Two triangles are congruent if one hypotenuse side and one of the perpendicular side of triangle is equal to the corresponding one hypotenuse side and one of the perpendicular side of the other triangle.

Fill in the Blanks Type Questions

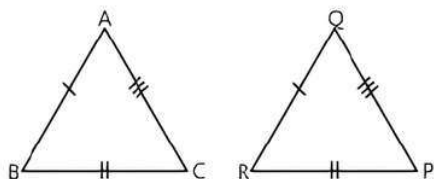
- Q 20. If two angles and the included side of one triangle are equal to two angles and the included sides of the other triangle, then the two triangles are
- Q 21. If $\triangle ABC \cong \triangle PQR$, then side of $\triangle PQR$ equal to the side BC of $\triangle ABC$ is
- Q 22. In $\triangle ABC$, $AB = AC$, $\angle A + \angle B + \angle C = 180^\circ$ and $\angle A = 50^\circ$, the measure of $\angle B$ is
- Q 23. In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then $\triangle ABC \cong$
- Q 24. In $\triangle ABC$, if $\angle C = \angle A$, then $AB =$

True/False Type Questions

- Q 25. If $\triangle ABC \cong \triangle PQR$, then $\angle B = \angle Q$ and $AC = PR$.
- Q 26. If ABC is isosceles triangle with $AB = AC$ and BD, CE are its two medians, then $BD = CE$.
- Q 27. In $\triangle ABC$, if $AB \perp BC$ and $\angle A = \angle C$, then $AB \neq AC$.
- Q 28. Every triangle is not congruent to itself.
- Q 29. The bisectors of two equal angles of a triangle are equal.

Solutions

- 1. (c) SSA
- 2. (b) Given $AB = QR$, $BC = PR$ and $CA = PQ$



$\therefore \triangle CBA \cong \triangle PRQ$

- 3. (c) In $\triangle ABD$ and $\triangle FEC$,
 $AB = EF$ [Given]
 $BC = DE$ [Given]
 $\Rightarrow BC + CD = DE + CD$
 $\Rightarrow BD = CE$
 and $\angle ABD = \angle CEF = 90^\circ$

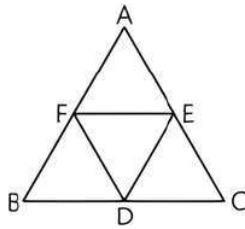
TRICK
 Two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, use SAS congruence rule.

$\therefore \triangle ABD \cong \triangle FEC$ [By SAS congruence]

- 4. (b) Given, $\triangle ABC \cong \triangle FDE$, $AB = 5$ cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$
 $\therefore \angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 80^\circ + 40^\circ + \angle C = 180^\circ \Rightarrow \angle C = 60^\circ$
 Since $\triangle ABC \cong \triangle FDE$ or $\triangle FDE \cong \triangle ABC$
 $\therefore FD = AB = 5$ cm
 $\angle E = \angle C = 60^\circ$
- 5. (b) In given $\triangle ABC$ and $\triangle DFE$,
 $AB = FD$
 $\angle A = \angle D$
 If we consider $AC = DE$, then we say that $\triangle ABC \cong \triangle DFE$ by SAS axiom.

COMMON ERROR
 Students are very careful for writing the congruency of two triangles (i.e., $\triangle ABC \cong \triangle PQR$). In this case we write all the angles corresponding to the equal angles, otherwise this will be wrong. So adequate practice is required.

- 6. (a) Since triangles, $\triangle DEF$, $\triangle AFE$, $\triangle CDE$ and $\triangle BDF$ are all congruents.
 Hence, $\triangle DEF$ is not congruent to $\triangle ABC$.



7. (b) In given $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$
 Since, $AB = AC$

TIP
 Angles opposite to equal sides of a triangle are equal.

$$\Rightarrow \angle C = \angle B$$

$$\therefore \angle C = 50^\circ$$

8. (c) In given $\triangle ABC$, $\angle B = 80^\circ$
 $BC = AB$

$$\Rightarrow \angle A = \angle C$$

[\therefore Angles opposite to equal sides of a triangle are equal]

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 80^\circ + \angle A = 180^\circ$$

$$\Rightarrow 2\angle A = 100^\circ$$

$$\Rightarrow \angle A = 50^\circ$$

9. (a) In given $\triangle PQR$, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm.
 Since, $\angle R = \angle P$

TIP
 Sides opposite to equal angles of a triangle are equal.

$$\Rightarrow PQ = QR$$

$$= 4 \text{ cm}$$

10. (b) Given $AB = AC$,
 $\Rightarrow \angle C = \angle B$ [\therefore Angles opposite to equal sides of a triangle are equal]

$$= 60^\circ$$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

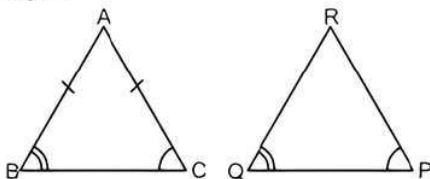
$$\Rightarrow \angle A + 60^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow \angle A = 60^\circ$$

$$\therefore \angle BAD = \frac{1}{2} \angle A$$
 [\therefore AD is the bisector of $\angle A$]

$$= \frac{60^\circ}{2} = 30^\circ$$

11. (a) In given $\triangle ABC$ and $\triangle PQR$
 $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$
 Since $AB = AC$, therefore $\triangle ABC$ is an isosceles triangle.



$$\therefore \angle C = \angle B$$

$$\Rightarrow \angle P = \angle Q$$

$\Rightarrow \triangle PQR$ is an isosceles triangle.
 But it is clear that $\triangle ABC$ is not congruent to $\triangle PQR$.

12. (a) In $\triangle ABC$,
 $AB = AC$ [Given]

TIP
 Angles opposite to equal sides of a triangle are equal.

$$\Rightarrow \angle ACB = \angle ABC$$

$$= 62^\circ$$

In $\triangle ABC$, use the sum of all angles of a triangle is 180°

$$\therefore \angle A + \angle B + \angle ACB = 180^\circ$$

$$\Rightarrow x + 62^\circ + 62^\circ = 180^\circ$$

$$\Rightarrow x = 180^\circ - 124^\circ = 56^\circ$$

13. (a) In the given figure, join PS.

In $\triangle PQS$ and $\triangle PRS$,

$$PQ = PR$$
 [Given]

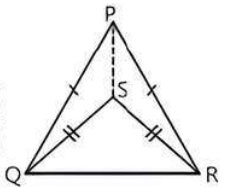
$$PS = PS$$
 [Common]

$$\text{and } QS = RS$$
 [Given]

$\therefore \triangle PQS \cong \triangle PRS$ by SSS congruency

$$\Rightarrow \angle PQS = \angle PRS$$

$$\Rightarrow \frac{\angle PQS}{\angle PRS} = \frac{1}{1}$$



14. (b) In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC$$
 [Given]

$$BD = CD$$
 [Given]

$$AD = AD$$
 [Common]

$\therefore \triangle ABD \cong \triangle ACD$ [By SSS]

$$\Rightarrow \angle BDA = \angle CDA$$
 [By CPCT]

$$\Rightarrow y = 60^\circ$$

$$\text{and } \angle BAD = \angle CAD$$
 [By CPCT]

$$35^\circ = x$$

$$\text{or } x = 35^\circ$$

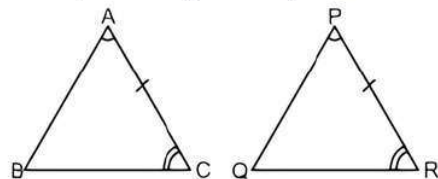
15. (a) **Assertion (A):** There is no rule of congruency when both triangles have equal corresponding angle but sides are different. So, these two triangles are not congruent.

Therefore, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

16. (a) **Assertion (A):** It is true to say that two angles and the including sides of two triangles are equal then they are congruent by ASA rule.



Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

17. (d) **Assertion (A):** In $\triangle PQR$,

$$PQ = QR$$

$$\Rightarrow \angle R = \angle P = 75^\circ$$

[\because Angles opposite to equal sides are equal]

So, Assertion (A) is false.

Reason (R): It is true to say that angles opposite to equal sides of triangle are equal.

Hence, Assertion (A) is false but Reason (R) is true.

18. (b) **Assertion (A):** In $\triangle PQB$ and $\triangle PRC$,

$$BP = PC \quad [\because P \text{ is the mid-point of } BC]$$

$$\angle Q = \angle R = 90^\circ$$

[\because Angles opposite to equal sides are equal]

and $PQ = PR$ [Given]

$$\therefore \triangle PQB \cong \triangle PRC \quad [\text{by RHS congruence}]$$

$$\Rightarrow BQ = CR \quad [\text{By CPCT}]$$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

19. (b) **Assertion (A):** In $\triangle PQR$ and $\triangle PSR$,

$$PQ = PS \quad [\text{Given}]$$

$$\angle RPQ = \angle RPS \quad [\because PR \text{ bisects } \angle P]$$

$$PR = PR \quad [\text{Common}]$$

$$\therefore \triangle PQR \cong \triangle PSR \quad [\text{by SAS congruence rule}]$$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. congruent.

21. Given $\triangle ABC \cong \triangle PQR$

$$\Rightarrow BC = QR \quad [\text{By CPCT}]$$

22. In given $\triangle ABC$, $AB = AC$ and $\angle A = 50^\circ$

$$\text{Since } AB = AC$$



TiP

Angles opposite to equal sides of a triangle are equal.

$$\Rightarrow \angle C = \angle B$$

$$\therefore \angle A + \angle B + \angle C = 180^\circ \quad [\text{Given}]$$

$$\therefore 50^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 130^\circ$$

$$\Rightarrow \angle B = 65^\circ.$$

23. $\triangle EFD$

24. In a $\triangle ABC$, if $\angle C = \angle A$, then sides opposite to equal angles are equal.

$$\therefore AB = BC$$

25. True,

$$\text{Given } \triangle ABC \cong \triangle PQR,$$

$$\therefore \angle B = \angle Q \text{ and } AC = PR \quad [\text{By CPCT}]$$

26. True,

Given, $\triangle ABC$ is isosceles.

$$\text{and } AB = AC$$

In $\triangle BAD$ and $\triangle CAE$

$$\angle A = \angle A \quad [\text{Common}]$$

$$AB = AC \quad [\text{Given}]$$

$$AD = AE$$

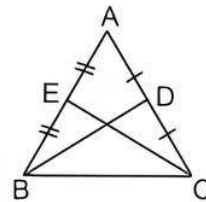
[\because D is mid-point of AC and E is mid-point of AB]

$$\therefore AD = DC \text{ and } AE = BE$$

$$\text{Also } AB = AC \Rightarrow AE = AD$$

$$\therefore \triangle BAD \cong \triangle CAE \quad [\text{By SAS congruence}]$$

$$\Rightarrow BD = CE \quad [\text{By CPCT}]$$



COMMON ERROR

Students make error while applying the congruence rule. So, please be careful.

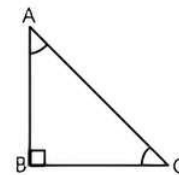
27. True,

$$\text{Since } AB \perp BC$$

$$\therefore \angle B = 90^\circ$$

In a right angled triangle, hypotenuse is always greater than any of the remaining sides.

$$\therefore AB \neq AC$$



28. False

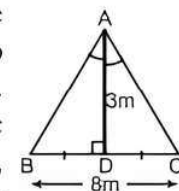
29. True



Case Study Based Questions

Case Study 1

One day, a geometry teacher came in the class and she decided to teach students about the triangle. She was considering a Mathematic problem for students; in $\triangle ABC$, AD is the perpendicular bisector of BC.



She asked the following questions from the students.

On the basis of the above information, solve the following questions.

Q 1. $\triangle ADB$ is congruent to $\triangle ADC$ by:

- a. AAS axiom b. ASA axiom
c. RHS axiom d. SAS axiom

Q 2. $\triangle ABC$ is a/an:

- a. right angled triangle
b. isosceles triangle
c. equilateral triangle
d. isosceles right angled triangle

Q 3. In given figure, length of AB in $\triangle ABC$ is:

- a. 6 m b. 4 m
c. 5 m d. 7 m

Q 4. The area of $\triangle ABC$ is:

- a. 9 m^2 b. 10 m^2
c. 12 m^2 d. 14 m^2

Q 5. Which of the following axiom is not satisfy the condition of congruency?

- a. RHS b. ASA
c. SSS d. SSA

Solutions

1. (d) In $\triangle ADB$ and $\triangle ADC$

$$AD = AD$$

[Common]



TIP

Perpendicular bisector divides the line in two equal parts.

$$BD = DC$$

[\because AD is perpendicular bisector of BC]

and $\angle ADB = \angle ADC = 90^\circ$

$\therefore \triangle ADB \cong \triangle ADC$ by SAS axiom.

So, option (d) is correct.

2. (b) $\because \triangle ADB \cong \triangle ADC$,

$$\therefore AB = AC$$

[By CPCT]

Hence, $\triangle ABC$ is an isosceles triangle.

So, option (b) is correct.

3. (c) In right angled $\triangle ADB$, use Pythagoras theorem

$$AB = \sqrt{(BD)^2 + (AD)^2} = \sqrt{(4)^2 + (3)^2}$$

$$\left[\because BD = DC = \frac{8}{2} = 4 \text{ m} \right]$$

[Common side]

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ m}$$

So, option (c) is correct.

4. (c) Area of $\triangle ABC = \frac{1}{2} \times BC \times AD$

$$= \frac{1}{2} \times 8 \times 3 = 12 \text{ m}^2$$

So, option (c) is correct.

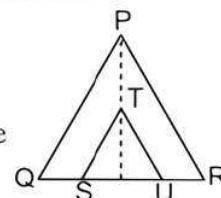
5. (d) SSA axiom is not the condition of congruency.

Case Study 2

Red Pyramid (or North Pyramid) is the largest pyramid in Cairo Egypt's capital. It is a rusty red pyramid made up of red limestones. The height of the pyramid is 105 m and its base is 220 m.



It is also believed that it was Egyptians first successfully attempt at constructing a smooth side pyramid. It can be seen that two triangles $\triangle PQR$ and $\triangle STU$ have been marked in the images of pyramid, such that $ST \parallel PQ$ and $TU \parallel PR$ and $\angle P = \angle T$.



On the basis of the above information, solve the following questions.

Q 1. Which of the following is not a criteria for congruency?

- a. SSS b. SSA c. ASA d. RHS

Q 2. If $\angle TSU = 40^\circ$ and $TS = TU$, then $\angle STU$ is:

- a. 100° b. 110°
c. 120° d. 130°

Q 3. If $\angle QPR = 50^\circ$ and $QP = PR$, then $\angle PQR$ is:

- a. 65° b. 70°
c. 75° d. 80°

Q 4. If hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangles are congruent by axiom.

- a. SSS b. RHS
c. SAS d. ASA

Q 5. Area of two congruent figures is:

- a. equal b. not equal
c. (a) or (b) d. do not say anything

Solutions

1. (b) SSA is not a criteria for congruency.

So, option (b) is correct.

2. (a) In $\triangle TSU$,

$$TS = TU$$

(Given)

\Rightarrow

$$\angle U = \angle S = 40^\circ$$

(Angles opposite to equal sides are equal)

\therefore

$$\angle T + \angle S + \angle U = 180^\circ$$

$$\angle T + 40^\circ + 40^\circ = 180^\circ$$

\Rightarrow

$$\angle T = 100$$

So, option (a) is correct.

3. (a) In $\triangle PQR$,

$$QP = PR$$

[Given]



TIP
Angles opposite to equal sides of a triangle are equal.

$$\begin{aligned} \Rightarrow \quad \angle R &= \angle Q \\ \therefore \quad \angle P + \angle Q + \angle R &= 180^\circ \\ \therefore \quad \angle P + \angle Q + \angle Q &= 180^\circ \\ 50^\circ + 2\angle Q &= 180^\circ \\ \Rightarrow \quad 2\angle Q &= 130^\circ \Rightarrow \angle Q = 65^\circ \end{aligned}$$

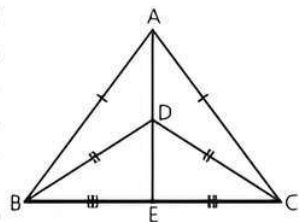
So, option (a) is correct.

4. (b) RHS
So, option (b) is correct.
5. (a) Area of two congruent figures is always equal.
So, option (a) is correct.

Case Study 3

Sunil is a farmer who is having a triangular plot. As the land is limited with him, so he decided to divide his land in four parts so that he can use multiple cropping method in which fields are growing with different seeds. The field is divided as shown below.

In this figure, $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC . Line AD is extended to BC , which intersect at point E .



On the basis of the above information, solve the following questions.

- Q 1.** If Sunil decides to grow crop in $\triangle ABD$, then which of the other triangle is of exact shape and size?
- a. $\triangle ABD \cong \triangle ACD$ b. $\triangle ABD \cong \triangle BDE$
c. $\triangle ABD \cong \triangle ACE$ d. $\triangle ACD \cong \triangle AEB$
- Q 2.** In $\triangle BDC$, if $\angle B + \angle D + \angle C = 180^\circ$, $\angle D = 70^\circ$, then $\angle B$ is equal to:
- a. 70° b. 60° c. 55° d. 80°
- Q 3.** If Sunil decides to grow multiple crop in $\triangle BDE$, then which of the other triangle is of exact shape and size?
- a. $\triangle BDE \cong \triangle CDE$ b. $\triangle BDE \cong \triangle CED$
c. $\triangle BED \cong \triangle CDE$ d. $\triangle CED \cong \triangle EBD$
- Q 4.** Which of the following pair is congruent?
- a. $\triangle ABE$ and $\triangle ADC$ b. $\triangle ABE$ and $\triangle ACE$
c. $\triangle AEC$ and $\triangle ADB$ d. $\triangle ABE$ and $\triangle ABD$
- Q 5.** Two circles with same radii but different centres, then they are:
- a. not always congruent
b. always congruent
c. congruent in some special cases
d. congruent when areas are different

Solutions

1. (a) In $\triangle ABD$ and $\triangle ACD$
- $AB = AC$ [Given]
 $BD = DC$ [Given]
 $AD = AD$ [Common]
Hence, $\triangle ABD \cong \triangle ACD$. [By SSS rule]
So, option (a) is correct.
2. (c) Given $\angle D = 70^\circ$
- $\therefore \angle B + \angle D + \angle C = 180^\circ$
 $\therefore \angle B + 70^\circ + \angle B = 180^\circ$
[$\because BD = DC \Rightarrow \angle C = \angle B$]
 $\Rightarrow 2\angle B = 110^\circ$
 $\Rightarrow \angle B = 55^\circ$
So, option (c) is correct.
3. (a) In $\triangle BDE$ and $\triangle CDE$,
- $BD = CD$ [Given]
 $DE = DE$ [Common]
and $BE = EC$ [Given]

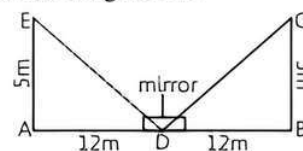
TRICK

Three sides of a triangle are equal to the three sides of another triangle, use SSS congruence rule.

- $\therefore \triangle BDE \cong \triangle CDE$ [By SSS rule]
So, option (a) is correct.
4. (b) In $\triangle ABE$ and $\triangle ACE$,
- $AB = AC$
 $BE = EC$
and $AE = AE$ [Common]
 $\therefore \triangle ABE \cong \triangle ACE$ [By SSS rule]
So, option (b) is correct.
5. (b) Two circles with same radii but different centres are always congruent.
So, option (b) is correct.

Case Study 4

John recently read a Mathematics experiment. He was keen to perform it on its own. He chose a long building whose height he want to know, he placed a mirror at ground. He is standing at some distance to the building as well as mirror. John height is 5 m and the distance of John from mirror is 12 m and distance of building from mirror is also 12 m and its height 5 m.



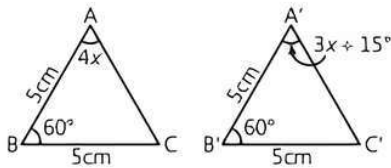
On the basis of the above information, solve the following questions.

- Q 1.** Write two congruent triangles formed in the given figure.
- Q 2.** Find the distance between top of building and mirror.



Q 3. Find the area of ΔAED .

Q 4. In the given figures, find the measure of $\angle B'A'C'$.



Solutions

1. In ΔAED and ΔBCD ,

$$AD = BD = 12 \text{ m}$$

$$AE = BC = 5 \text{ m}$$

and $\angle A = \angle B = 90^\circ$

$$\therefore \Delta AED \cong \Delta BCD \quad [\text{By SAS rule}]$$

2. In right angled ΔEAD , use Pythagoras theorem,

$$\begin{aligned} ED &= \sqrt{(AE)^2 + (AD)^2} \\ &= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ m.} \end{aligned}$$

Hence, the distance between top of building and mirror is 13 m.

3. Area of $\Delta AED = \frac{1}{2} \times AD \times AE$

$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$$

4. In ΔABC and $\Delta A'B'C'$

$$AB = A'B' = 5 \text{ cm} \quad [\text{Given}]$$

$$\angle B = \angle B' = 60^\circ \quad [\text{Given}]$$

and $BC = B'C' = 5 \text{ cm}$

$$\therefore \Delta ABC \cong \Delta A'B'C' \quad [\text{By SAS congruence}]$$

$$\Rightarrow \angle BAC = \angle B'A'C' \quad [\text{By CPCT}]$$

$$\Rightarrow 4x = 3x + 15^\circ$$

$$\Rightarrow x = 15^\circ$$

$$\therefore \angle B'A'C' = 3x + 15^\circ$$

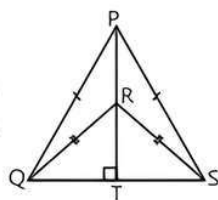
$$= 3 \times 15^\circ + 15^\circ$$

$$[\because x = 15^\circ]$$

$$= 45^\circ + 15^\circ = 60^\circ$$

Case Study 5

Kushal is a property dealer. He has been in this business from past 12 yr. He bought a triangular shaped plot. It divides the plot into four parts which is shown in the figure.



On the basis of the above information, solve the following questions.

Q 1. If Sunil decides to sell two plots, then show that two triangles ΔPQR and ΔPSR have exact shape and size?

Q 2. In ΔQRS , $\angle RQS = 50^\circ$, then find $\angle QSR$.

Q 3. If $\angle QPS = 60^\circ$, then find $\angle QPT$.

Solutions

1. In ΔPQR and ΔPSR ,

$$PQ = PS \quad [\text{Given}]$$

$$QR = RS \quad [\text{Given}]$$

and $PR = PR \quad [\text{Common}]$

$$\therefore \Delta PQR \cong \Delta PSR \quad [\text{By SSS congruency}]$$

2. In ΔQRS , $\angle RQS = 50^\circ$

Since $QR = RS$

$$\Rightarrow \angle QSR = \angle RQS = 50^\circ$$

$$[\because \text{angles opposite to equal sides are equal}]$$

3. In ΔQPT and ΔSPT ,

$$PQ = PS \quad [\text{Given}]$$

$$PT = PT \quad [\text{Common}]$$

and $\angle QTP = \angle STP = 90^\circ$

TRICK

The hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, use RHS congruence rule.

$$\therefore \Delta QPT \cong \Delta SPT \quad [\text{By RHS Congruence rule}]$$

$$\Rightarrow \angle QPT = \angle SPT \quad [\text{By CPCT}]$$

$$\therefore \angle QPS = 60^\circ$$

$$\Rightarrow 2\angle QPT = 60^\circ \quad \Rightarrow \angle QPT = 30^\circ$$



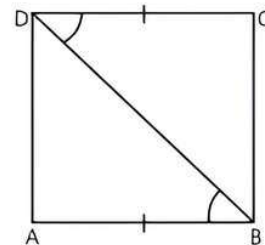
Very Short Answer Type Questions

Q 1. It is given that $\Delta ABC \cong \Delta DEF$. Is it true to say that $AB = EF$? Justify your answer.

Q 2. In triangle ABC and PQR , $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that two triangles are congruent? Give reason for your answer.

Q 3. E and F are respectively the mid-points of equal sides AB and AC of ΔABC , show that $BF = CE$.

Q 4. In the figure, if $AB = DC$, $\angle ABD = \angle CDB$ which congruence rule would you apply to prove $\Delta ABD \cong \Delta CDB$?



Q 5. $\Delta PQR \cong \Delta ABC$, if $\angle P + \angle Q + \angle R = 180^\circ$, $PQ = 5 \text{ cm}$, $\angle Q = 40^\circ$ and $\angle P = 80^\circ$, calculate the value of $\angle C$.

Q 6. In ΔABC , $\angle C = \angle A$ and $BC = 6 \text{ cm}$. If $AC = 5 \text{ cm}$, then find the length of AB .

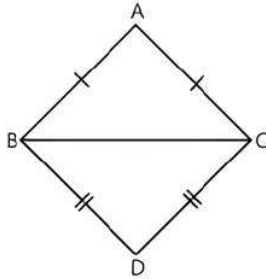
Q 7. In ΔPQR , PE is the perpendicular bisector of $\angle QPR$, then prove that $PQ = PR$.

Q 8. In $\triangle PQR$, $\angle P + \angle Q + \angle R = 180^\circ$, $\angle P = 70^\circ$ and $\angle R = 30^\circ$. Which angle of this triangle is the longest?

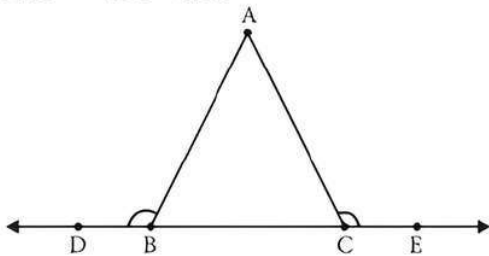


Short Answer Type-I Questions \blacktriangledown

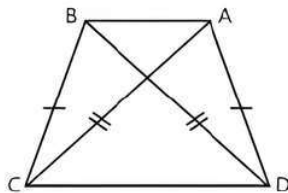
Q 1. In the figure, $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC . Prove that $\angle ABD = \angle ACD$.



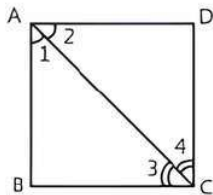
Q 2. In the figure, if $\angle ABD = \angle ACE$, then prove that $AB = AC$.



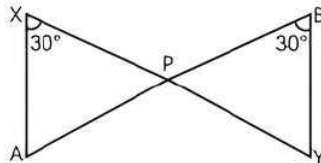
Q 3. In given figure, $AD = BC$ and $BD = AC$, prove $\angle DAB = \angle CBA$.



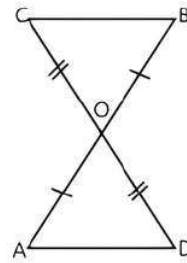
Q 4. In given figure, if $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove that $BC = CD$.



Q 5. In figure, $XP = BP$ and $AX \parallel BY$, prove that $\triangle APX \cong \triangle YPB$.



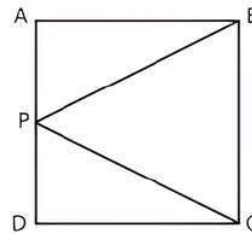
Q 6. In the figure, $OA = OB$ and $OD = OC$. Show that
(i) $\triangle AOD \cong \triangle BOC$
(ii) $AD = BC$



Q 7. In triangle ABC , D is mid-point of side AC such that $BD = \frac{1}{2}AC$ and $\angle A + \angle B + \angle C = 180^\circ$.

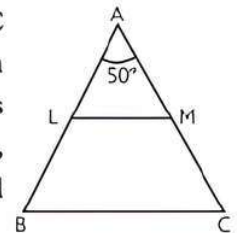
Show that $\angle ABC$ is right angle.

Q 8. In figure, $ABCD$ is a square and P is the mid-point of AD . BP and CP are joined. Prove that $\angle PCB = \angle PBC$.



Q 9. PS is an altitude of an isosceles triangle PQR in which $PQ = PR$. Show that PS bisects $\angle P$.

Q 10. In the given figure, ABC is an isosceles triangle in which $AB = AC$ and LM is parallel to BC . If $\angle A = 50^\circ$, $\angle A + \angle B + \angle C = 180^\circ$ find $\angle ABC$.



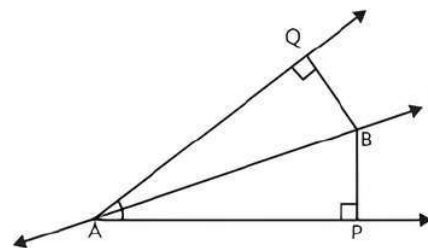
Q 11. Two lines l and m intersect at point O and P is a point on line n passing through point O such that P is equidistant from l and m . Prove that n is the bisector of the angle formed by l and m .



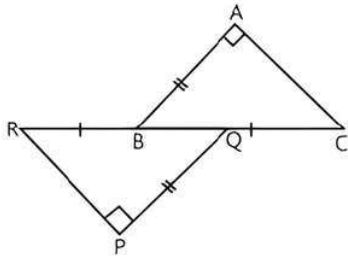
Short Answer Type-II Questions \blacktriangledown

Q 1. Line l is the bisector of an angle A and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure). Show that:

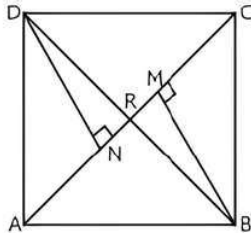
- (i) $\triangle APB \cong \triangle AQB$.
- (ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.



Q 2. In the given figure, $BA \perp CA$, $RP \perp QP$, $AB = PQ$ and $BR = CQ$. Prove that $PR = AC$.

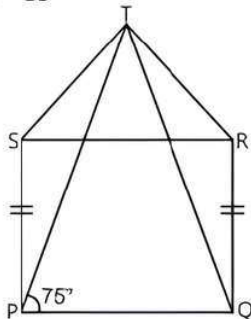


Q 3. In the given figure, BM and DN are both parallel and perpendicular to AC such that AC bisects BD and $BM = DN$. Prove that $MR = NR$.



Q 4. In the given figure, $PQRS$ is a square and SRT is an equilateral triangle prove that:

- (i) $PT = QT$
- (ii) $\angle TQR = 15^\circ$

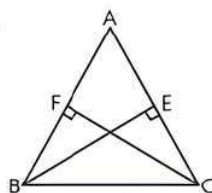


Q 5. In a right-angled triangle, if one acute angle is double the other, then prove that the hypotenuse is double the smallest side.

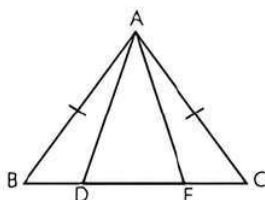
Q 6. In $\triangle ABC$, altitudes BE and CF are equal.

Prove the following results:

- (i) $\triangle ABE \cong \triangle ACF$
- (ii) $AB = AC$

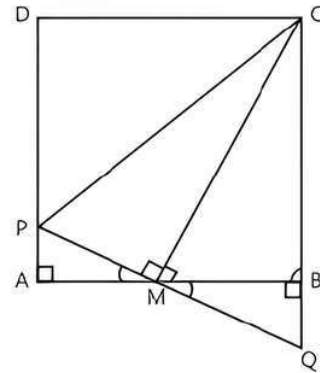


Q 7. In the given figure, ABC is an isosceles triangle with $AB = AC$, D and E are points on BC such that $BE = CD$. Show that ADE is an isosceles triangle.



Q 8. $ABCD$ is a square. M is the mid-point of AB and $PQ \perp CM$, PQ meets AD at P , CB produced at Q . Prove that:

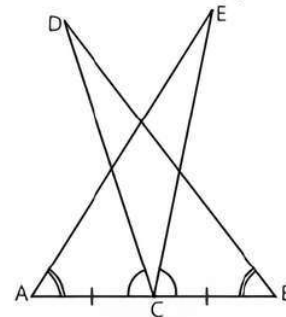
- (i) $PA = BQ$
- (ii) $CP = AB + PA$



Q 9. Triangle ABC is an isosceles triangle such that $AB = AC$. Side BA is produced to D such that $AD = AB$. Show that $\angle BCD$ is a right angle.

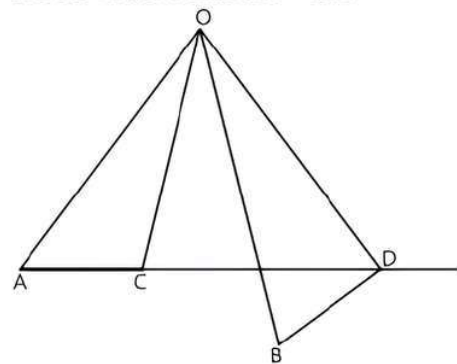
Long Answer Type Questions

Q 1. In the given figure, if $AC = BC$, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$, then prove that $AE = BD$.



Q 2. In a rhombus $ABCD$, O is any interior point such that $OA = OC$. Then prove that D , O and B are collinear.

Q 3. In the figure, $OA = OB$, $OC = OD$ and $\angle AOB = \angle COD$. Prove that $AC = BD$.

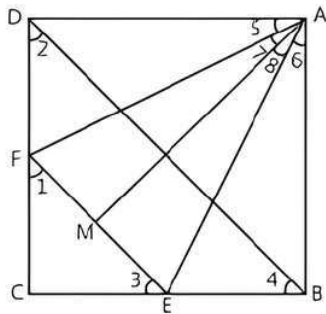


Q 4. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Q 5. If two isosceles triangle have a common base prove that the line joining their vertices bisects them at right angles.

Q 6. In the given figure, ABCD is a square, EF is parallel to diagonal BD, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ and $EM = FM$. Prove that:

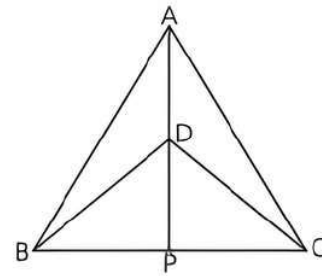
- (i) $DF = BE$.
 (ii) AM bisects $\angle BAD$.



Q 7. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that:

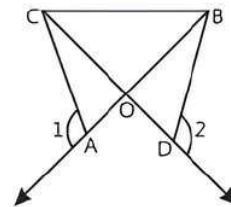
- (i) $\triangle ABD \cong \triangle ACD$
 (ii) $\triangle ABP \cong \triangle ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$.



Q 8. Line segment joining mid-point M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that $AD = BC$.

Q 9. In the given figure, $OA = OD$ and $\angle 1 = \angle 2$. Prove that $\triangle OCB$ is an isosceles triangle.



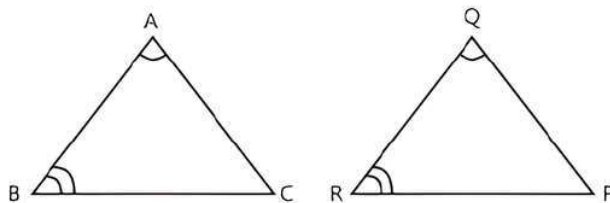
Solutions

Very Short Answer Type Questions

1. No, AB and EF are not corresponding sides in triangles ABC and DEF but AB corresponds to DE. So, $AB = DE$.

2. In $\triangle ABC$ and $\triangle QRP$,

$$\begin{aligned} \angle A &= \angle Q && \text{[Given]} \\ \angle B &= \angle R && \text{[Given]} \end{aligned}$$



If $AB = QR$, then,

$\triangle ABC$ will be congruent to $\triangle QRP$.

i.e., $\triangle ABC \cong \triangle QRP$ [By ASA congruence]

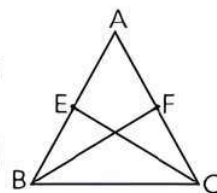
3. In $\triangle ABF$ and $\triangle ACE$,

$$AB = AC \quad \text{[Given]}$$

$$\angle A = \angle A$$

[Common]

$$AF = AE$$



$$[\because AB = AC \Rightarrow \frac{AB}{2} = \frac{AC}{2} \Rightarrow AE = AF]$$

$\therefore \triangle ABF \cong \triangle ACE$ [By SAS congruence]

$\Rightarrow BF = CE$ [By CPCT]

4. In $\triangle ABD$ and $\triangle CDB$,

$$AB = DC \quad \text{[Given]}$$

$$\angle ABD = \angle CDB \quad \text{[Given]}$$

$$BD = BD \quad \text{[Common]}$$

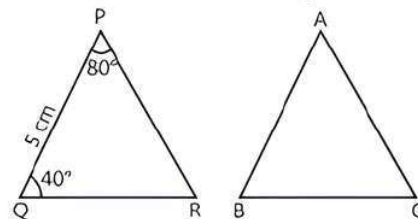
\therefore By SAS congruence,

$$\triangle ABD \cong \triangle CDB$$

5. Given, $PQ = 5$ cm, $\angle Q = 40^\circ$ and $\angle P = 80^\circ$.

$$\therefore \angle P + \angle Q + \angle R = 180^\circ$$

$$\therefore \angle R = 180^\circ - (\angle P + \angle Q)$$



$$\Rightarrow \angle R = 180^\circ - (80^\circ + 40^\circ)$$

$$\Rightarrow \angle R = 60^\circ$$

$\therefore \triangle PQR \cong \triangle ABC$ [Given]

$\therefore \angle R = \angle C = 60^\circ$ [By CPCT]

6. In $\triangle ABC$, $\angle C = \angle A$



TIP

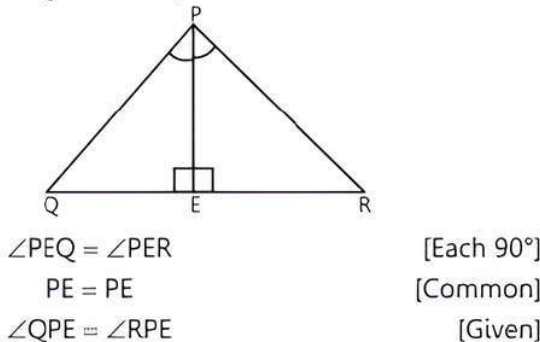
Sides opposite to equal angles of a triangle are equal.

$$\Rightarrow AB = BC$$

$\Rightarrow AB = BC = 6$ cm [Given]

Hence, the length of AB is 6 cm.

7. In $\triangle PEQ$ and $\triangle PER$,



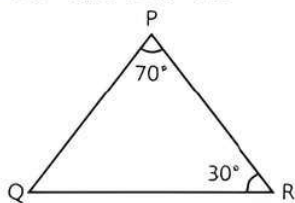
TRICK

Two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, use ASA congruence rule.

$\therefore \triangle PEQ \cong \triangle PER$ [By ASA congruence]
 Thus, $PQ = PR$ [By CPCT]
Hence proved

8. Given, in $\triangle PQR$,

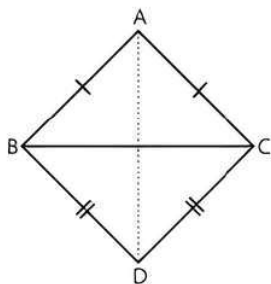
$\angle P = 70^\circ$ and $\angle R = 30^\circ$



$\angle P + \angle Q + \angle R = 180^\circ$
 $\Rightarrow \angle Q = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$
 Hence, $\angle Q$ is greatest.

Short Answer Type-I Questions

1. Join AD in the figure.



In $\triangle ABD$ and $\triangle ACD$,
 $AB = AC$ [Given]
 $BD = CD$ [Given]
 $AD = AD$ [Common]
 $\therefore \triangle ABD \cong \triangle ACD$ [By SSS congruence]
 Thus, $\angle ABD = \angle ACD$ [By CPCT]
Hence proved

2. From figure, $\angle ABD + \angle ABE = 180^\circ$

[Linear pair] ... (1)
 Also, $\angle ACE + \angle ACD = 180^\circ$ [Linear pair] ... (2)
 $\angle ABD = \angle ACE$ [Given] ... (3)
 From eqs. (1), (2) and (3), we get
 $\angle ABE = \angle ACD$
 or $\angle ABC = \angle ACB$



TIP

Sides opposite to equal angles of a triangle are equal.

$\Rightarrow AC = AB$

Hence proved

3. In $\triangle DAB$ and $\triangle CBA$,

$AD = BC$ [Given]
 $BD = AC$ [Given]
 $AB = AB$ [Common]
 $\therefore \triangle DAB \cong \triangle CBA$ [By SSS congruence]
 Thus, $\angle DAB = \angle CBA$ [By CPCT]

Hence proved

4. In $\triangle ABC$ and $\triangle ADC$,

$\angle 1 = \angle 2$ [Given]
 $AC = AC$ [Common]
 $\angle 3 = \angle 4$ [Given]
 $\therefore \triangle ABC \cong \triangle ADC$ [By ASA congruence]
 Thus, $BC = CD$ [By CPCT]

Hence proved

5. Given, $AX \parallel BY$

In $\triangle APX$ and $\triangle YPB$,
 $PX = BP$ [Given]
 $\angle PXA = \angle PBY = 30^\circ$
 $\angle XPA = \angle BPY$ [Vertically opposite angles]
 $\therefore \triangle APX \cong \triangle YPB$

Hence proved

6. (i) In $\triangle AOD$ and $\triangle BOC$,

$OA = OB$ [Given]
 $\angle AOD = \angle BOC$ [Vertically opposite angles]
 $OD = OC$ [Given]
 $\therefore \triangle AOD \cong \triangle BOC$ [By SAS congruence]

(ii) $\therefore \triangle AOD \cong \triangle BOC$

$\therefore AD = BC$ [By CPCT]

Hence proved

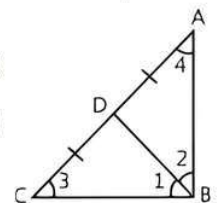
7. In $\triangle ABC$, D is mid-point of AC.

$\therefore AD = CD = \frac{1}{2} AC$... (1)

$BD = \frac{1}{2} AC$ [Given] ... (2)

From eqs. (1) and (2), we get

$AD = BD$ and $CD = BD$



TIP

Angles opposite to equal sides of a triangle are equal.

$\Rightarrow \angle 2 = \angle 4$ and $\angle 1 = \angle 3$... (3)

In $\triangle ABC$,

$\angle ABC + \angle ACB + \angle CAB = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 1 + \angle 2 = 180^\circ$ [From eq. (3)]

$\Rightarrow 2(\angle 1 + \angle 2) = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

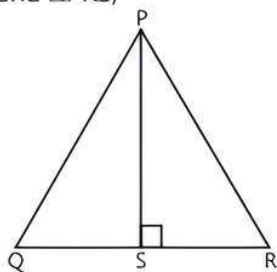
$\Rightarrow \angle ABC = 90^\circ$

So, $\triangle ABC$ is a right angle.

Hence proved

8. In $\triangle PAB$ and $\triangle PDC$,
 $PA = PD$ [Given]
 $AB = CD$ [Sides of a square]
 $\angle PAB = \angle PDC$ [Each 90°]
 $\therefore \triangle PAB \cong \triangle PDC$ [By SAS congruence]
 Thus, $PB = PC$ [By CPCT]
 $\Rightarrow \angle PCB = \angle PBC$ [Angles opposite to equal sides of a triangle are equal]
Hence proved

9. In $\triangle PQS$ and $\triangle PRS$,



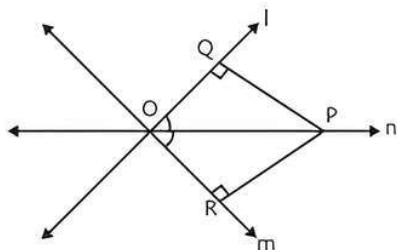
- $PQ = PR$ [Given]
 $PS = PS$ [Common]
 $\angle PSQ = \angle PSR = 90^\circ$ [PS is altitude]

TR!CK

The hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, use RHS congruence rule.

- $\therefore \triangle PQS \cong \triangle PRS$ [By RHS congruence]
 Thus, $\angle QPS = \angle RPS$ [By CPCT]
 Hence, PS bisects $\angle P$. **Hence proved**
10. In $\triangle ABC$, $AB = AC$ [Given]
 $\therefore \angle C = \angle B$
 [Angles opposite to equal sides are equal]
 $\therefore \angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 50^\circ + \angle B + \angle C = 180^\circ$
 $\Rightarrow 50^\circ + 2\angle B = 180^\circ$
 $\Rightarrow \angle B = \frac{180^\circ - 50^\circ}{2} = 65^\circ$
 $\therefore \angle B = \angle C = 65^\circ$
 Hence, $\angle ABC = 65^\circ$

- 11.



- In $\triangle OQP$ and $\triangle ORP$,
 $\angle PQO = \angle PRO$ [Each 90°]
 $OP = OP$ [Common]
 $OQ = OR$ [Given]
 $\therefore \triangle OQP \cong \triangle ORP$ [By RHS congruence]
 Thus, $\angle POQ = \angle POR$ [By CPCT]
 Hence, n is the bisector of $\angle QOR$.
Hence proved

Short Answer Type-II Questions

1. **Given:** l bisects $\angle A$, $BQ \perp AQ$ and $BP \perp AP$.

To Prove: (i) $\triangle APB \cong \triangle AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

Proof: (i) In $\triangle APB$ and $\triangle AQB$,

$\angle PAB = \angle QAB$ [l is the bisector of $\angle A$]

$\angle APB = \angle AQB$ [Each 90°]

$AB = AB$ [Common]

$\therefore \triangle APB \cong \triangle AQB$ [By AAS congruence]

(ii) $\therefore \triangle APB \cong \triangle AQB$

$BP = BQ$ [By CPCT]

or B is equidistant from the arms of $\angle A$.

Hence proved

2. **Given:** $BA \perp CA$, $RP \perp QP$, $AB = PQ$ and $BR = CQ$.

To Prove: $AC = PR$

Proof: Since $BR = CQ$

Adding BQ on both sides, we get

$BR + BQ = CQ + BQ$

$\Rightarrow QR = BC$... (1)

Now, in $\triangle QPR$ and $\triangle BAC$

$PQ = AB$ [Given]

$\angle P = \angle A$ [Each 90°]

$QR = BC$ [From eq. (1)]

$\therefore \triangle QPR \cong \triangle BAC$ [By RHS congruence]

Thus, $PR = AC$ [By CPCT]

Hence proved

3. **Given:** $BM \perp AC$, $DN \perp AC$ and $BM = DN$.

To Prove: AC bisects BD .

Proof: In $\triangle BMR$ and $\triangle DNR$,

$\angle N = \angle M$ [Each 90°]

$BM = DN$ [Given]

$BR = DR$ [\because BD is a bisector of AC]

$\triangle BMR \cong \triangle DNR$ [By RHS congruence]

$\Rightarrow MR = NR$ [By CPCT]

4. **Given:** $PQRS$ is a square and SRT is an equilateral triangle.

To Prove: (i) $PT = QT$

(ii) $\angle TQR = 15^\circ$

Proof: (i) \because $PQRS$ is a square and SRT is an equilateral triangle.

$\therefore \angle PSR = 90^\circ$ and $\angle TSR = 60^\circ$

$\angle PSR + \angle TSR = 150^\circ$

Similarly, $\angle QRT = 150^\circ$

In $\triangle PST$ and $\triangle QRT$,

$PS = QR$ [Sides of a square]

$\angle PST = \angle QRT = 150^\circ$ [Proved above]

$ST = RT$

[Sides of an equilateral triangle]

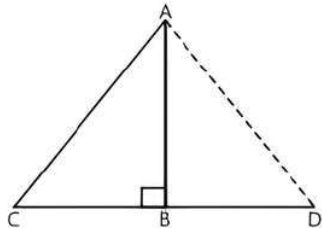
$\therefore \triangle PST \cong \triangle QRT$ [By SAS congruence]

Thus, $PT = QT$ [By CPCT] **Hence proved**

(ii) In $\triangle TPQ$,
 $PT = QT$ [Proved above]
 $\Rightarrow \angle PQT = \angle QPT = 75^\circ$
 Since PQRS is a square.
 Therefore,
 $\angle PQR = 90^\circ$
 $\Rightarrow \angle PQT + \angle TQR = 90^\circ$
 $\Rightarrow 75^\circ + \angle TQR = 90^\circ$
 $\Rightarrow \angle TQR = 15^\circ$

5. Let us consider a right-angled $\triangle ABC$ right-angled at B.

Let $\angle CAB = x$ and $\angle ACB = 2x$.
 Produce CB to D such that $BD = BC$.



In $\triangle ABC$ and $\triangle ABD$,
 $AB = AB$ [Common]
 $BC = BD$ [Produced]
 $\angle ABC = \angle ABD$ [Each 90°]
 $\therefore \triangle ABC \cong \triangle ABD$ [By SAS congruence]
 $\therefore AC = AD$ and $\angle CAB = \angle DAB$ [by CPCT]
 Now, $\angle CAD = x + x = 2x$
 $\Rightarrow \angle CAD = \angle ACB$



TIP

Sides opposite to equal angles of a triangle are equal.

$\therefore AD = CD$
 or $AD = 2BC$
 $\Rightarrow AC = 2BC$

Hence, hypotenuse AC is double the smallest side BC.

6. **Given:** In $\triangle ABC$, $BE \perp AC$, $CF \perp AB$ and $BE = CF$

To Prove: (i) $\triangle ABE \cong \triangle ACF$ (ii) $AB = AC$

Proof: (i) In $\triangle ABE$ and $\triangle ACF$,
 $\angle AEB = \angle AFC = 90^\circ$
 $\angle BAE = \angle CAF$ [Common angle]
 and $BE = CF$ [Given]
 $\therefore \triangle ABE \cong \triangle ACF$ [By AAS congruence]
 (ii) Since $\triangle ABE \cong \triangle ACF$
 $\therefore AB = AC$ [By CPCT]

Hence proved

7. **Given:** In $\triangle ABC$, $AB = AC$ and $BE = CD$.

To Prove: $\triangle ADE$ is an isosceles triangle.

Proof: In $\triangle ABC$,
 $AB = AC$ [Given]
 $\angle C = \angle B$ [Angles opposite to equal sides of a triangle]
 $BE = CD$ [Given]

$\Rightarrow BD + DE = DE + EC$

$\Rightarrow BD = EC$

In $\triangle ABD$ and $\triangle ACE$,

$AB = AC$ [Given]

$\angle B = \angle C$ [Proved above]

$BD = EC$ [Proved above]

$\therefore \triangle ABD \cong \triangle ACE$ [By SAS congruence]

Thus, $\angle ADB = \angle AEC$ [By CPCT]

Also, $\angle ADB + \angle ADE = \angle AEC + \angle AED = 180^\circ$

[Linear pair]

$\Rightarrow \angle ADE = \angle AED$ [$\because \angle ADB = \angle AEC$]

Thus, $\triangle ADE$ is an isosceles triangle. **Hence proved**

8. **Given:** ABCD is a square. M is the mid-point of AB and $PQ \perp CM$.

To Prove: (i) $PA = BQ$

(ii) $CP = AB + PA$

Proof:

(i) In $\triangle AMP$ and $\triangle BMQ$,

$\angle PAM = \angle QBM$ [Each 90°]

$AM = BM$ [\because M is mid-point of AB]

$\angle AMP = \angle BMQ$

[Vertically opposite angles]

$\therefore \triangle AMP \cong \triangle BMQ$ [By ASA congruence]

Thus, $PM = QM$ and $PA = BQ$ [By CPCT]

Hence proved

(ii) In $\triangle CMP$ and $\triangle CMQ$,

$CM = CM$ [Common]

$\angle PMC = \angle QMC$ [Each 90°]

$PM = QM$ [Proved above]

$\therefore \triangle CMP \cong \triangle CMQ$ [By SAS congruence]

Thus, $CP = CQ$ [By CPCT]

$\Rightarrow CP = CB + BQ$

$\Rightarrow CP = AB + BQ$ [$\because CB = AB$]

$\Rightarrow CP = AB + PA$ [$\because BQ = PA$]

Hence proved

9. **Given:** In $\triangle ABC$, $AB = AC$ and $AD = AB$.

Construction: Join CD

To Prove: $\angle BCD$ is a right angle.

Proof: In $\triangle ABC$, $AC = AB$

$\therefore \angle 1 = \angle 2$... (1)

[angles opposite to equal sides are equal]

In $\triangle ADC$, $AB = AD$

$\therefore AC = AD$ [$\because AB = AC$]

$\angle 4 = \angle 3$... (2)

In $\triangle BCD$,

[Angles opposite to equal sides are equal]

Now, $\angle 1 + (\angle 2 + \angle 3) + \angle 4 = 180^\circ$

[Angle sum property of a triangle]

$\Rightarrow \angle 2 + (\angle 2 + \angle 3) + \angle 3 = 180^\circ$

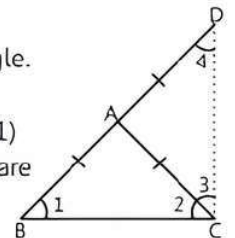
[From eq. (1) and eq. (2)]

$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ$

$\therefore \angle BCD$ is a right angled.

Hence proved



Long Answer Type Questions

1. **Given:** $AC = BC$, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$.

To Prove: $BD = AE$

Proof: Since $\angle DCA = \angle ECB$

Adding $\angle DCE$ on both sides, we get

$$\angle DCA + \angle DCE = \angle ECB + \angle DCE$$

$$\Rightarrow \angle ECA = \angle DCB$$

In $\triangle ACE$ and $\triangle BCD$,

$$\angle ECA = \angle DCB \quad [\text{Proved above}]$$

$$\angle EAC = \angle DBC \quad [\text{Given}]$$

$$AC = BC \quad [\text{Given}]$$

$\therefore \triangle ACE \cong \triangle BCD$ [by ASA congruence]

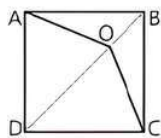
Thus, $AE = BD$ [By CPCT]

Hence proved

2. **Given:** In rhombus $ABCD$, $OA = OC$

To Prove: D, O and B are collinear.

Construction: Join OD and OB .



Proof: In $\triangle AOB$ and $\triangle COB$,

$$AO = CO \quad [\text{Given}]$$

$$OB = OB \quad [\text{Common}]$$

$$AB = BC \quad [\because ABCD \text{ is a rhombus}]$$

$\therefore \triangle AOB \cong \triangle COB$ [By SSS congruence]

Thus, $\angle AOB = \angle COB$... (1) [By CPCT]

Similarly, $\triangle AOD \cong \triangle COD$

Thus, $\angle AOD = \angle COD$... (2) [By CPCT]

But $\angle AOD + \angle COD + \angle COB + \angle AOB = 360^\circ$
[Complete angle]

$$\Rightarrow 2(\angle AOD + \angle AOB) = 360^\circ$$

[From eqs. (1) and (2)]

$$\Rightarrow \angle AOD + \angle AOB = 180^\circ$$

$\therefore D, O$ and B are collinear. **Hence proved**

3. **Given:** $OA = OB$, $OC = OD$ and $\angle AOB = \angle COD$.

To Prove: $AC = BD$

Proof: Since $\angle AOB = \angle COD$

Subtracting $\angle COB$ on both sides, we get

$$\angle AOB - \angle COB = \angle COD - \angle COB$$

$$\angle AOC = \angle BOD$$

Now, in $\triangle AOC$ and $\triangle BOD$,

$$AO = OB \quad [\text{Given}]$$

$$\angle AOC = \angle BOD \quad [\text{Proved above}]$$

$$OC = OD \quad [\text{Given}]$$

TRICK

Two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, use SAS congruence rule.

$\therefore \triangle AOC \cong \triangle BOD$ [By SAS congruence]

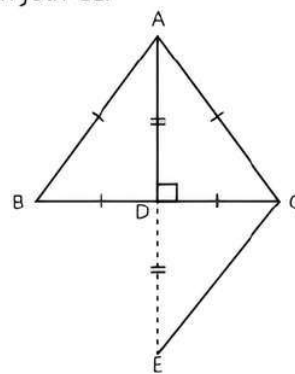
Thus, $AC = BD$ [By CPCT]

Hence proved

4. Let us consider a $\triangle ABC$ where AD bisects $\angle A$ and side BC .

To Prove: $\triangle ABC$ is isosceles triangle.

Construction: Produce AD to E such that $AD = DE$ and then join CE .



Proof: In $\triangle ABD$ and $\triangle ECD$,

$$BD = CD \quad [\text{Given}]$$

$$\angle ADB = \angle EDC \quad [\text{Vertically opposite angles}]$$

$$AD = ED \quad [\text{By construction}]$$

$\therefore \triangle ABD \cong \triangle ECD$ [By SAS congruence]

Thus, $AB = CE$ [By CPCT] ... (1)

$$\angle BAD = \angle CED \quad [\text{By CPCT}] \dots (2)$$

$$\angle BAD = \angle CAD \quad [\text{Given}] \dots (3)$$

From eqs. (2) and (3), we get

$$\angle CAD = \angle CED$$

$\therefore EC = AC$... (4)

[sides opposite to equal angles are equal]

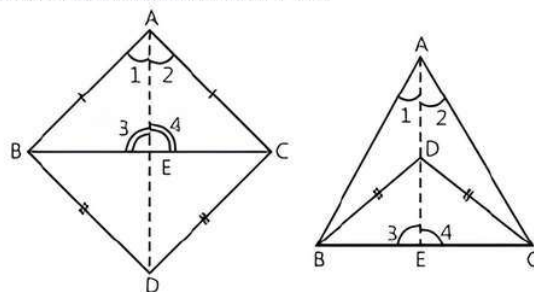
From eqs. (1) and (4), we get

$$AB = AC$$

So, the triangle is isosceles.

Hence proved

5. Let us consider two isosceles triangles ABC and BDC with common base BC .



To Prove: AD bisects BC at right angles.

Proof: In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common}]$$

$$BD = CD \quad [\text{Given}]$$

$\therefore \triangle ABD \cong \triangle ACD$ [By SSS congruence]

Thus, $\angle 1 = \angle 2$ [By CPCT]

Now, in $\triangle ABE$ and $\triangle ACE$,

$$AB = AC \quad [\text{Given}]$$

$$\angle 1 = \angle 2 \quad [\text{Proved above}]$$

$$AE = AE \quad [\text{Common}]$$

$\therefore \triangle ABE \cong \triangle ACE$ [By SAS congruence]

Thus, $BE = CE$

and $\angle 3 = \angle 4$ [By CPCT]
 But $\angle 3 + \angle 4 = 180^\circ$ [Linear pair]
 $\Rightarrow 2\angle 3 = 180^\circ$
 $\Rightarrow \angle 3 = 90^\circ$
 $\therefore \angle 3 = \angle 4 = 90^\circ$
 Hence, AD bisects BC at right angles.

Hence proved

6. **Given:** ABCD is a square, EF || BD and EM = FM.

To Prove: (i) DF = BE.

(ii) AM bisects \angle BAD.

Proof:

(i) Since EF || BD [Given]
 $\Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Given]
 $\angle 2 = \angle 4$ [Each 45°]
 $\Rightarrow \angle 1 = \angle 3$
 $\therefore CE = CF$ [Sides opposite to equal angles of a triangle are equal]
 $\Rightarrow BC - CE = BC - CF$ [Subtracting from BC both sides]
 $\Rightarrow BC - CE = CD - CF$
 $[\because BC = CD, \text{ sides of a square}]$

$\Rightarrow BE = DF$

or $DF = BE$

Hence proved

(ii) In $\triangle ADF$ and $\triangle ABE$,

AD = AB [Sides of a square]

$\angle D = \angle B$ [Each 90°]

DF = BE [Proved above]

$\therefore \triangle ADF \cong \triangle ABE$ [By SAS congruence]

Thus, AF = AE and $\angle 5 = \angle 6$ [By CPCT] ... (1)

In $\triangle AMF$ and $\triangle AME$,

AF = AE [From eq. (1)]

AM = AM [Common]

FM = EM [Given]

$\therefore \triangle AMF \cong \triangle AME$ [By SSS congruence]

Thus, $\angle 7 = \angle 8$... (2)

Adding eqs. (1) and (2), we get

$\angle 5 + \angle 7 = \angle 6 + \angle 8$

$\angle DAM = \angle BAM$

\therefore AM bisects \angle BAD.

Hence proved

7. **Given:** $\triangle ABC$ and $\triangle DCB$ are two isosceles triangles, in which AB = AC and BD = DC.

To Prove:

(i) $\triangle ABD \cong \triangle ACD$ (ii) $\triangle ABP \cong \triangle ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$.

Proof: (i) In $\triangle ABD$ and $\triangle ACD$,

AB = AC [Given]

BD = DC [Given]

and AD = AD [Common side]

$\therefore \triangle ABD \cong \triangle ACD$ [By SSS congruence]

(ii) In $\triangle ABP$ and $\triangle ACP$,

AB = AC [Given]

AP = AP [Common side]

and $\angle PAB = \angle PAC$ [As $\triangle ABD \cong \triangle ACD$]

$\Rightarrow \angle BAD = \angle CAD$

$\therefore \triangle ABP \cong \triangle ACP$ [By SAS congruence]

(iii) As prove above,

$\triangle ABD \cong \triangle ACD$... (1)

$\Rightarrow \angle BAD = \angle CAD$ [By CPCT]

and $\triangle ABP \cong \triangle ACP$... (2)

On subtracting eq. (1) from eq. (2),

$\triangle ABP - \triangle ABD \cong \triangle ACP - \triangle ACD$

$\Rightarrow \triangle BDP \cong \triangle CDP$

$\therefore \angle BDP = \angle CDP$ [By CPCT]

Hence, AP bisects $\angle A$ as well as $\angle D$.

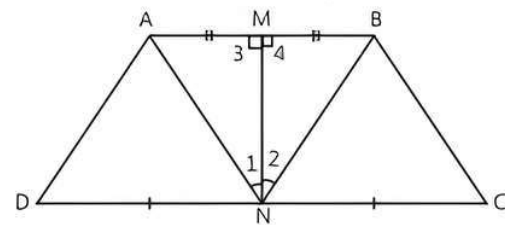
Hence proved

8. **Given:** In trapezium ABCD, AB || DC, M and N are mid-points of AB and DC respectively.

To Prove: AD = BC

Construction: Join AN, MN and BN,

Proof: In $\triangle AMN$ and $\triangle BMN$,



AM = MB [M is the mid-point of AB]

$\angle 3 = \angle 4$ [Each 90°]

MN = MN [Common]

$\therefore \triangle AMN \cong \triangle BMN$ [By SAS congruence]

Thus, $\angle 1 = \angle 2$ and AN = BN [By CPCT]

Subtracting both the sides from 90° , we get

$90^\circ - \angle 1 = 90^\circ - \angle 2$ [$\because MN \perp DC$]

$\Rightarrow \angle AND = \angle BNC$... (1)

In $\triangle ADN$ and $\triangle BCN$,

AN = BN [Proved above]

$\angle AND = \angle BNC$, [From eq. (1)]

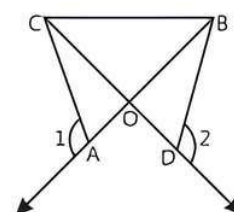
DN = NC [\because N is mid-point of CD]

$\therefore \triangle ADN \cong \triangle BCN$ [By SAS congruence]

Thus, AD = BC [By CPCT]

Hence proved

9. **Given:** OA = OD and $\angle 1 = \angle 2$



To Prove: $\triangle OCB$ is an isosceles triangle.

Proof: From figure,

$$\angle 1 + \angle CAB = 180^\circ \quad \text{[Linear pair]}$$

$$\text{and } \angle 2 + \angle BDC = 180^\circ \quad \text{[Linear pair]}$$

$$\therefore \angle 1 + \angle CAB = \angle 2 + \angle BDC$$

$$\text{But } \angle 1 = \angle 2 \quad \text{[Given]}$$

$$\therefore \angle CAB = \angle BDC$$

In $\triangle OAC$ and $\triangle ODB$,

$$\angle CAB = \angle BDC \quad \text{[Proved above]}$$

$$OA = OD \quad \text{[Given]}$$

$$\angle AOC = \angle DOB \quad \text{[Vertically opposite angles]}$$

$$\therefore \triangle OAC \cong \triangle ODB \quad \text{[By ASA congruence]}$$

$$\text{Thus, } OC = OB \quad \text{[By CPCT]}$$

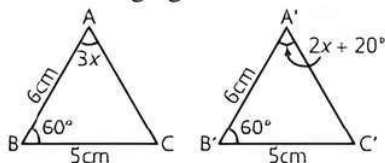
$\therefore \triangle OCB$ is an isosceles triangle. **Hence proved**



Chapter Test

Multiple Choice Questions

Q 1. In the following figures, measure of $\angle B'A'C'$ is:



- a. 50° b. 60° c. 40° d. 65°

Q 2. In triangles ABC and RQP , if $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$, then two triangles are:

- a. isosceles but not necessarily congruent
b. isosceles and congruent
c. congruent but not isosceles
d. neither congruent nor isosceles.

Assertion and Reason Type Questions

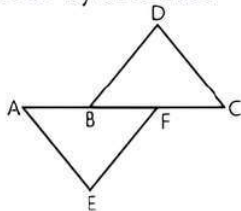
Directions (Q. Nos. 3-4) In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): In $\triangle ABC$, if $AB = AC$ and $\angle B = 75^\circ$, then $\angle C = 75^\circ$.

Reason (R): In a triangle, angles opposite to equal sides are equal.

Q 4. Assertion (A): In the given figure, if $AB = CF$, $EF = BD$ and $\angle AFE = \angle CBD$, then $\triangle AFE \cong \triangle CBD$ by SAS rule.



Reason (R): If two sides and included angle of one triangle is equal to the two sides and

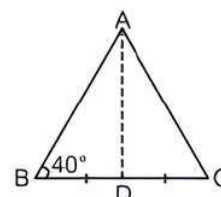
included angle of the other triangle then triangles are congruent by SAS rule.

Fill in the Blanks

- Q 5. The number of triangles that can be drawn with the measure of each angle less than 60° , is
- Q 6. Triangles ABC and PQR satisfy the relations $AB = QP$, $\angle B = \angle P$ and $BC = PR$, then these triangles are congruent by

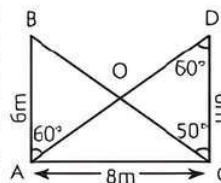
True/False

- Q 7. The medians of an equilateral triangle are equal.
- Q 8. ABC is an isosceles triangle such that $AB = AC$ and AD is the median to base BC . Then $\angle BAD = 50^\circ$



Case Study Based Questions

Q 9. Aryan is studying in class IX. He observe two equal height of poles of equal length $AB = CD = 6\text{m}$. These poles are 8m apart from each other and O is the point of intersection of the lines joining the top of each pole to the foot of opposite poles.



On the basis of the above information, solve the following questions.

- (i) In which congruent rule, $\triangle ABC \cong \triangle CDA$?
(ii) Find the length of BC .

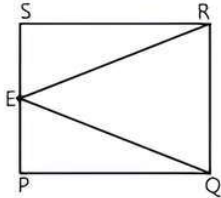
OR

What is the area of $\triangle ACD$?

- (iii) Prove that $\triangle ABO \cong \triangle DCO$.

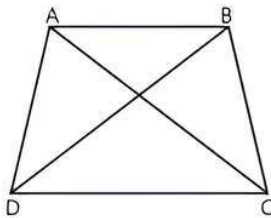
Very Short Answer Type Questions

- Q 10. In $\triangle ABC$ and $\triangle PQR$, it is given that $\angle A = \angle R$, $\angle C = \angle P$ and $\angle B = \angle Q$. Find either both triangles are isosceles or congruent.
- Q 11. In the adjoining figure, PQRS is a square and E is the mid-point of PS and joined ER and EQ. Prove that $EQ = ER$.



Short Answer Type-I Questions

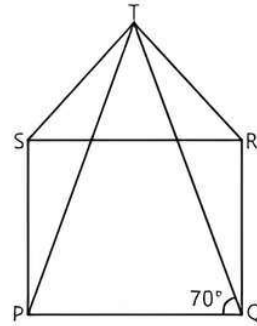
- Q 12. In the adjoining figure, $AD = BC$ and $BD = CA$ prove that $\angle ADB = \angle BCA$.



- Q 13. CE and BF are perpendiculars to AB and AC respectively in $\triangle ABC$ such that $BE = CF$. Prove that $\angle B = \angle C$.

Short Answer Type-II Questions

- Q 14. In right angled $\triangle ABC$, $\angle C = 90^\circ$. M is the mid point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that:
(i) $\triangle AMC \cong \triangle BMD$
(ii) $\angle DBC = \angle ACB$
- Q 15. In the adjoining figure, PQRS is a square and SRT is an equilateral triangle. Prove that
(i) $PT = QT$
(ii) $\angle TPS = 20^\circ$



Long Answer Type Question

- Q 16. If ABC is a right angled triangle such that $AB = AC$ and bisector of angle C intersects the side AB at D, then prove that $AC + AD = BC$.